Lecture: 3-1 Derivatives of Polynomials and Exponentials

Derivative of a Constant Function: $\frac{d}{dx}(c) =$ __________

Example 1: Find the derivatives of the following functions.

(a) $f(x) = 5.4$

(b) $g(x) = \pi^7$

(c) $h(x) = \ln 2$

Example 2: Using the definition of the derivative, find the derivatives of the following functions.

(a) $f(x) = x^2$

(b) $f(x) = x^3$

The Power Rule: If n is a positive integer, then $\frac{d}{dx}x^n =$ __________

Example 3: Find the derivatives of the following functions.

(a) $f(x) = x^9$

(b) $y = x^{99}$

(c) $\dfrac{d}{dt}(t^5)$

> Using the definition of the derivative you can prove that the following derivatives. Does the power rule appear to hold for non-integer exponents as well?
>
> (a) $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$
>
> (b) $\dfrac{d}{dx}\sqrt{x} = \dfrac{1}{2\sqrt{x}}$

Example 4: Differentiate the following functions.

(a) $f(x) = \dfrac{1}{x^5}$

(b) $y = \sqrt[3]{x^5}$

> Using the power rule we can find equations of tangent lines much more quickly! We can also find the **normal line**, which is defined as the line through a point P that is perpendicular to the tangent line at P.

Example 5: Find equations of the tangent line and normal line to the curve $y = x^2\sqrt{x}$ at the point $(1, 1)$.

The Constant Multiple Rule: If c is a constant and f is differentiable function then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x).$$

Example 6: Differentiate the following functions.

(a) $\frac{d}{dx}(5x^7)$

(b) $\frac{d}{dx}(-3\sqrt{x^5})$

The Sum/Difference Rule: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x).$$

Example 7: Find the derivative of $y = x^7 + 10x^3 - 7x^2 + 2x - 9$.

Example 8: Find the points on the curve $y = x^4 - 2x^2 + 4$ where the tangent line is horizontal.

Example 9: Find the derivatives of the following functions.

(a) $y = (5x^2 - 2)^2$

(b) $f(x) = \dfrac{\sqrt{x} + 2x - 3}{x^3}$

Derivative of the Natural Exponential Function: $\dfrac{d}{dx}e^x = e^x$

Example 10: Find the derivatives of the following functions.

(a) $f(t) = \sqrt{3t} + \sqrt{\dfrac{3}{t}}$

(b) $f(x) = e^{x+2} + 4$

Example 11: At what point on the curve $y = e^x$ is the tangent line parallel to the line $y - 5x = 2$?

Example 13: Biologists have proposed a cubic function to model the length L of an Alaskan rockfish at age A:

$$L = 0.0155A^3 - 0.372A^2 + 3.95A + 1.21$$

where L is measured in inches and A in years. Calculate $\frac{dL}{dA}$ at $A = 12$ and interpret your answer.

Example 14: The equation of motion of a particle is $s = 2t^3 - 15t^2 + 36t + 1$. Find the velocity and acceleration functions. Then, determine the acceleration when the velocity is zero.

Example 15: Find the following limits.

(a) $\lim_{h \to 0} \frac{(2+h)^5 - 32}{h}$

(b) $\lim_{x \to 1} \frac{x^{99} - 1}{x - 1}$